MOMENT COEFFICIENTS FOR CONTINUOUS RIGID FRAMES WITH PARABOLIC MEMBERS

By

ROBERT JOE LARKIN

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

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Thesis Approved:

Thesis Adviser

Dean of the Graduate School

PREFACE

The author wishes to express his indebtedness and gratitude to the following persons.

To Professor Jan J. Tuma, Head of the School of Civil Engineering, for his invaluable aid, his constructive criticism, and for acting as major adviser in the preparation of this thesis.

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R. J. L.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

M	End moment
RM	Moment due to rotation
FM	Fixed end moment
FH	Fixed end reaction
Q	End moment coefficient
L	Length of span
H	Horizontal thrust of the girder
V	Shear
P	Unbalance of shears at a joint
I	Moment of inertia of the column
I_c	Moment of inertia at the center of the girder
a	Column parameter
b	Girder parameter
θ	Angular rotation
ϕ	Angle of inclination of the girder
Δ	Horizontal displacement
K	Stiffness factor
D	Distribution factor
C	Carry-over factor

SIGN CONVENTION

Moments

Horizontal shears and thrusts

Angular rotations

Horizontal displacements

PART I

INTRODUCTION

Tables of end moments due to a unit force applied at joints of two-, three-, and four-span continuous rigid frames are presented in this thesis. Each frame is comprised of second degree parabolic girders and straight columns hinged at the base. The girder rise (bL) and the column length (aL) are expressed as functions of the span length (L). The frames are analyzed for all combinations of the parameters "a" and "b" that are usually encountered in practice.

The moment coefficients and the solutions of the matrices were obtained by use of the IBM 650 Digital Computer.

The sign convention and nomenclature used in this thesis conforms with those usually accepted as standard.

This thesis is a part of a study of continuous rigid frames supervised by Professor Jan J. Tuma. Three studies were completed in this series. Gillespie investigated continuous frames with bent members and constant moments of inertia, loaded by uniformly distributed load.[2] Carmen investigated continuous frames with curved members and constant moments of inertia, loaded by uniformly distributed load.[1] Gillespie extended his first study to the investigation of end moments in gabled frames with constant moments of inertia, due to unit force applied at joints.[3] This work is the extension of the idea presented in Reference 3 to the frames with curved members.

PART II

DEFORMATION EQUATIONS

1. Slope-Deflection Equations

A typical joint of a continuous rigid frame with straight columns, parabolic girders, and a unit load at the joint (j) is considered (Fig. 2-1). The column is of constant cross-section and is hinged at the bottom. The girders are equal in size and their moments of inertia vary as the secant of the slope of the girder. The moment of inertia of the column is equal to the central moment of inertia of the girder.

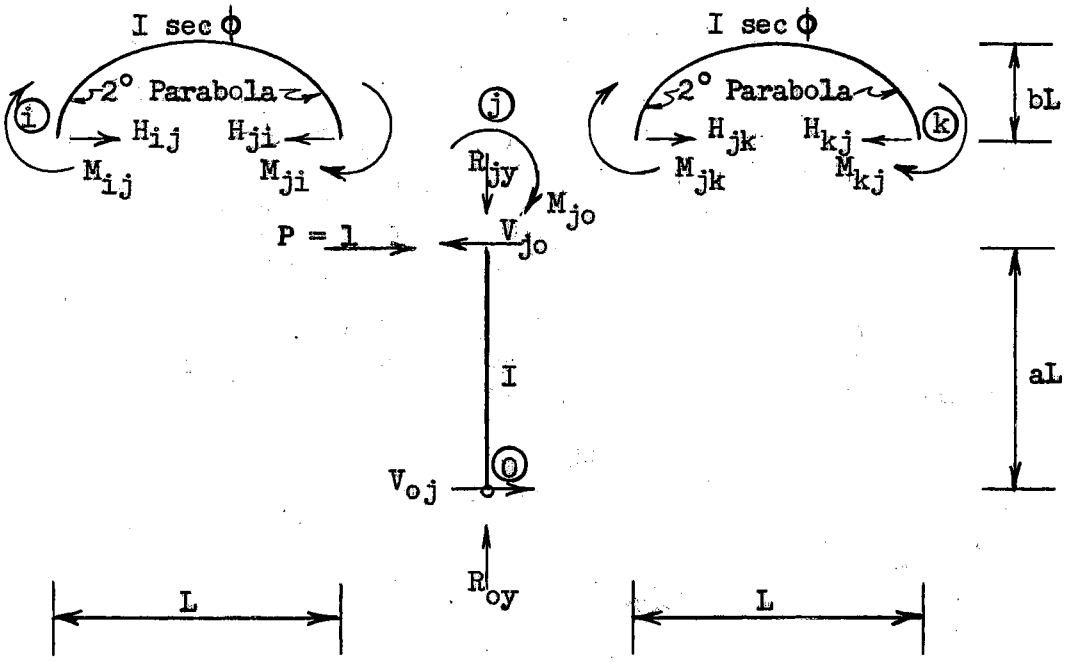


Figure 2-1

A Typical Joint of a Continuous Rigid Frame

The moment slope deflection equations for the members shown in Fig. (2-1) are:

$$M_{ji} = \frac{9\,EI_c}{L}\theta_j - \frac{3\,EI_c}{L}\theta_i - \frac{7.5\,EI_c}{bL^2}\Delta_{ijx} + FM_{ji}$$

$$M_{jo} = \frac{3\,EI}{aL}\theta_j - \frac{3\,EI}{a^2L^2}\Delta_{jx} + FM_{jo} \qquad (2\text{-}1)$$

$$M_{jk} = \frac{9\,EI_c}{L}\theta_j - \frac{3\,EI}{L}\theta_k - \frac{7.5\,EI_c}{bL^2}\Delta_{jkx} + FM_{jk}$$

The shear and thrust equations for the same members are:

$$H_{ij} = H_{ji} = \frac{7.5\,EI_c}{bL^2}\left[\theta_i - \theta_j + \frac{3\Delta_{ijx}}{2\,bL}\right] + FH_{ij}$$

$$H_{kj} = H_{jk} = \frac{7.5\,EI_c}{bL^2}\left[\theta_j - \theta_k + \frac{3\Delta_{jkx}}{2\,bL}\right] + FH_{kj} \qquad (2\text{-}2)$$

$$V_{jo} = V_{oj} = \frac{3\,EI}{a^2L^2}\theta_j - \frac{3\,EI}{a^3L^3}\Delta_{jx} + FH_{jo}$$

Similar equations may be stated for each joint of this frame.

2. Equilibrium Equations

The following equations of static equilibrium can be written for the joint (j) (Fig. 2-1):

$$\Sigma M_j = 0 = M_{ji} + M_{jo} + M_{jk} \qquad (2\text{-}3)$$

$$\Sigma F_{jx} = 0 = P + V_{jo} + H_{ji} - H_{jk} \qquad (2\text{-}4)$$

$$\Sigma F_{jy} = 0 \qquad (2\text{-}5)$$

Equations (2-3), (2-4), and (2-5) can be written for each joint of this frame.

No vertical displacement of joints is considered in this thesis; therefore, Equation (2-5) is not necessary for the solution of moment coefficients due to a unit load at a joint.

3. Moment Coefficients

Equations (2-3) and (2-4) are used in obtaining the moment coefficients due to a unit load at a joint.

The matrix for a unit load at a joint of a continuous rigid frame is quite large and its solution is difficult and laborious. Therefore, the matrix is divided into two cases: Case II - symmetrical load, and Case I - antisymmetrical load.

After solving these two independent cases, the final moment coefficients are obtained by superposition.

4. Matrix Tables

The matrices for Case I and Case II for a unit load applied at the first joint of a two-, three-, and four-span frame are shown in Tables (2-1), (2-2), and (2-3). If the unit load is applied at any other joint, the new matrices may be formulated from Tables (2-1), (2-2), and (2-3) by displacing the load factor to the new position in the matrix.

5. Solution

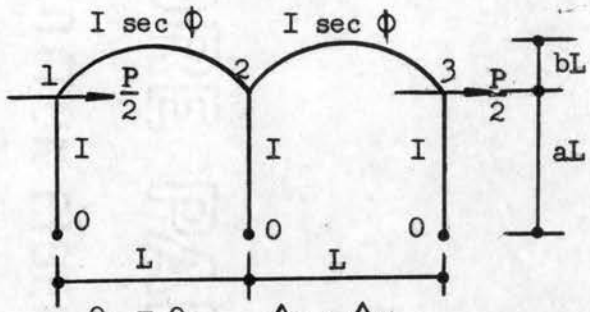
The solution of the matrices for Case I and Case II was accomplished by using the IBM 650 Digital Computer. The Fortransit

Method was used in the programming.

The use of Fortransit programming eliminated most manual calculations with the exception of adding the results of Case I and Case II on an adding machine to obtain the moment coefficients.

TABLE 2-1 MATRIX FOR TWO SPAN FRAME WITH PARABOLIC GIRDER

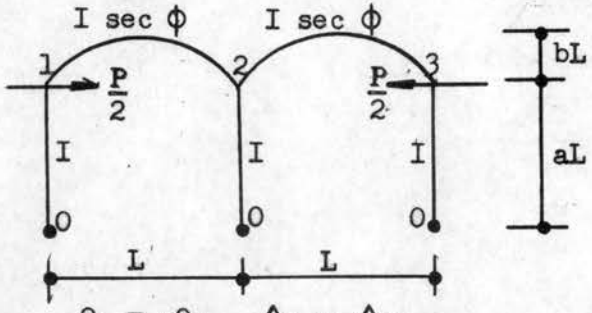
Case I



Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI_c}{L^2}\Delta_1$	$\frac{EI_c}{L^2}\Delta_2$	L
$\Sigma M_1 = 0$	$\frac{3}{a} + 9$	-3	$\frac{7.5}{b} - \frac{3}{a^2}$	$-\frac{7.5}{b}$	
$\Sigma M_2 = 0$	-6	$\frac{3}{a} + 18$	$-\frac{15}{b}$	$\frac{15}{b} - \frac{3}{a^2}$	
$\Sigma H_{1x} = 0$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{7.5}{b}$	$-\frac{3}{a^3} - \frac{11.25}{b^2}$	$\frac{11.25}{b^2}$	.500
$\Sigma H_{2x} = 0$	$\frac{15}{b}$	$\frac{3}{a^2} - \frac{15}{b}$	$\frac{22.5}{b^2}$	$\frac{-3}{a^3} - \frac{22.5}{b^2}$	

Case II

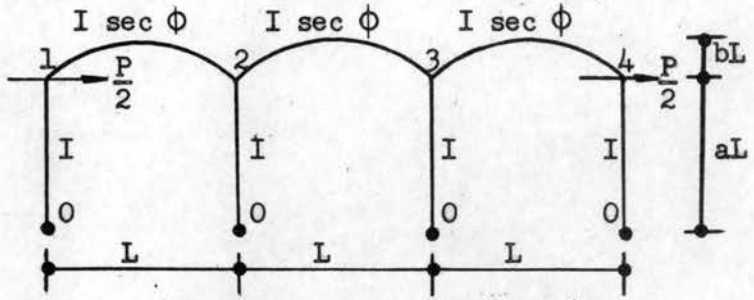


Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L^2}\Delta_1$	L
$\Sigma M_1 = 0$	$\frac{3}{a} + 9$	$\frac{7.5}{b} - \frac{3}{a^2}$	
$\Sigma H_{1x} = 0$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{-3}{a^3} - \frac{11.25}{b^2}$	.500

TABLE 2-2 MATRIX FOR THREE SPAN FRAME WITH PARABOLIC GIRDER

CASE I

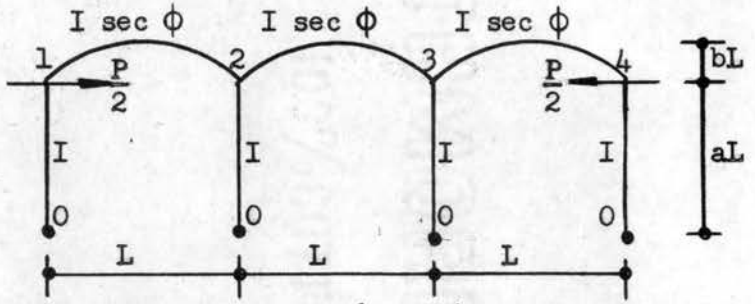


$\theta_1 = \theta_4 \quad \theta_2 = \theta_3 \quad \Delta_1 = \Delta_4 \quad \Delta_2 = \Delta_3$

Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI_c}{L^2}\Delta_1$	$\frac{EI_c}{L^2}\Delta_2$	L
$\Sigma M_1 = 0$	$\frac{3}{a} + 9$	-3	$\frac{7.5}{b} - \frac{3}{a^2}$	$-\frac{7.5}{b}$	
$\Sigma M_2 = 0$	-3	$15 + \frac{3}{a}$	$-\frac{7.5}{b}$	$\frac{7.5}{b} - \frac{3}{a^2}$	
$\Sigma H_{1x} = 0$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{7.5}{b}$	$-\frac{11.25}{b^2} - \frac{3}{a^3}$	$\frac{11.25}{b^2}$	.500
$\Sigma H_{2x} = 0$	$\frac{7.5}{b}$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{11.25}{b^2}$	$-\frac{11.25}{b^2} - \frac{3}{a^3}$	

CASE II



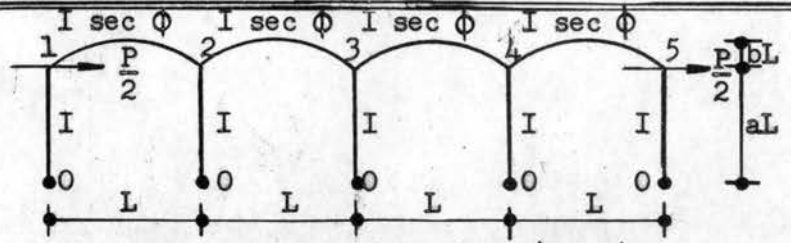
$\theta_1 = -\theta_4 \quad \theta_2 = -\theta_3 \quad \Delta_1 = -\Delta_4 \quad \Delta_2 = -\Delta_3$

Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI_c}{L^2}\Delta_1$	$\frac{EI_c}{L^2}\Delta_2$	L
$\Sigma M_1 = 0$	$\frac{3}{a} + 9$	-3	$\frac{7.5}{b} \quad \frac{-3}{a^2}$	$-\frac{7.5}{b}$	
$\Sigma M_2 = 0$	-3	$21 + \frac{3}{a}$	$-\frac{7.5}{b}$	$\frac{22.5}{b} \quad \frac{-3}{a^2}$	
$\Sigma H_{1x} = 0$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{7.5}{b}$	$-\frac{11.25}{b^2} \quad \frac{-3}{a^3}$	$\frac{11.25}{b^2}$	.500
$\Sigma H_{2x} = 0$	$\frac{7.5}{b}$	$\frac{3}{a^2} - \frac{22.5}{b}$	$\frac{11.25}{b^2}$	$-\frac{33.75}{b^2} \quad \frac{-3}{a^3}$	

TABLE 2-3 MATRIX FOR FOUR SPAN FRAME WITH PARABOLIC GIRDER

CASE I

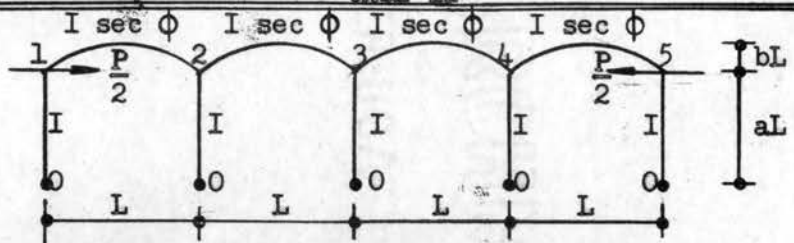


$\theta_1 = \theta_5 \quad \theta_2 = \theta_4 \quad \Delta_1 = \Delta_5 \quad \Delta_2 = \Delta_4$

Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI}{L}\theta_3$	$\frac{EI_c}{L^2}\Delta_1$	$\frac{EI_c}{L^2}\Delta_2$	$\frac{EI_c}{L^2}\Delta_3$	L
$\Sigma M_1 = 0$	$\frac{3}{a} + 9$	-3		$\frac{7.5}{b} - \frac{3}{a^2}$	$-\frac{7.5}{b}$		
$\Sigma M_2 = 0$	-3	$18 + \frac{3}{a}$	-3	$-\frac{7.5}{b}$	$\frac{15}{b} - \frac{3}{a^2}$	$-\frac{7.5}{b}$	
$\Sigma M_3 = 0$		-6	$18 + \frac{3}{a}$		$-\frac{15}{b}$	$\frac{15}{b} - \frac{3}{a^2}$	
$\Sigma H_{1x} = 0$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{7.5}{b}$		$-\frac{11.25}{b^2} - \frac{3}{a^3}$	$\frac{11.25}{b^2}$		.500
$\Sigma H_{2x} = 0$	$\frac{7.5}{b}$	$\frac{3}{a^2} - \frac{15}{b}$	$\frac{7.5}{b}$	$\frac{11.25}{b^2}$	$-\frac{22.5}{b^2} - \frac{3}{a^3}$	$\frac{11.25}{b^2}$	
$\Sigma H_{3x} = 0$		$\frac{15}{b}$	$\frac{3}{a^2} - \frac{15}{b}$		$\frac{22.5}{b^2}$	$-\frac{22.5}{b^2} - \frac{3}{a^3}$	

CASE II



$\theta_1 = -\theta_5 \quad \theta_2 = -\theta_4 \quad \Delta_1 = -\Delta_5 \quad \Delta_2 = -\Delta_4$

Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI_c}{L^2}\Delta_1$	$\frac{EI_c}{L^2}\Delta_2$	L
$\Sigma M_1 = 0$	$\frac{3}{a} + 9$	-3	$\frac{7.5}{b} - \frac{3}{a^2}$	$-\frac{7.5}{b}$	
$\Sigma M_2 = 0$	-3	$18 + \frac{3}{a}$	$-\frac{7.5}{b}$	$\frac{15}{b} - \frac{3}{a^2}$	
$\Sigma H_{1x} = 0$	$\frac{3}{a^2} - \frac{7.5}{b}$	$\frac{7.5}{b}$	$-\frac{11.25}{b^2} - \frac{3}{a^3}$	$\frac{11.25}{b^2}$	.500
$\Sigma H_{2x} = 0$	$\frac{7.5}{b}$	$\frac{3}{a^2} - \frac{15}{b}$	$\frac{11.25}{b^2}$	$-\frac{22.5}{b^2} - \frac{3}{a^3}$	

PART III

TABLES

1. General Notes

The tables for the moment coefficients are divided into the following three groups:

Group 3-2 Two Span Frames

Group 3-3 Three Span Frames

Group 3-4 Four Span Frames

The number of each table consists of three terms. The first term is the number of the section in which that table occurs, the second term is the number of spans per frame used in obtaining the moment coefficients, and the third term is the joint to which the unit load is applied.

The tables are comprised of several major parts, namely:

1. Table title and number.

2. Illustration of the frame (A figure containing symbols which designate a joint, length, load, and moment of inertia of a member).

3. Column parameter (a).

4. Girder parameter (b).

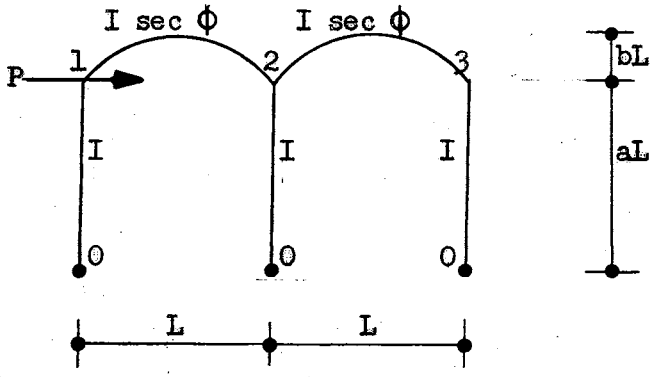
5. Moment coefficients (Q).

6. End moment equation ($M_{ij} = PQ_{ij}L$).

2. Steps of Procedure

1. Calculate parameters "a" and "b."
2. Compute moments due to rotation and end moments due to load.
   (a) Calculate stiffness factors, distribution factors, and the carry-over factors.
   (b) Calculate the fixed end moments due to loads with joints fixed against translation.
   (c) Distribute the fixed end moments to obtain the moments due to rotation and end moments due to load.
3. Calculate the unbalance of shears at joints (P_i).
4. Select the moment coefficients (Q_{ij}).
5. Compute the end moments due to unbalance of shears at joints ($M_{ij} = P_i Q_{ij} L$).
6. Compute the final end moments by adding the moments computed in Steps 2 and 5.

TWO SPAN FRAME PARABOLIC GIRDER TABLE 3-2-1



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.08833	-.11379	-.13102	-.14299	-.15164
0.4	-.14148	-.17171	-.19826	-.22082	-.23978
0.5	-.16849	-.19798	-.22550	-.25029	-.27223
0.6	-.19618	-.22433	-.25167	-.27734	-.30096
0.8	-.25335	-.27845	-.30394	-.32914	-.35355
1.0	-.31247	-.33473	-.35785	-.38132	-.40476

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.01167	-.00594	-.01523	-.02056	-.02387
0.4	+.04809	+.01718	-.00496	-.02082	-.03229
0.5	+.06726	+.03254	+.00562	-.01508	-.03100
0.6	+.08643	+.04900	+.01833	-.00654	-.02660
0.8	+.12438	+.08345	+.04754	+.01632	-.01062
1.0	+.16167	+.11867	+.07941	+.04388	+.01190

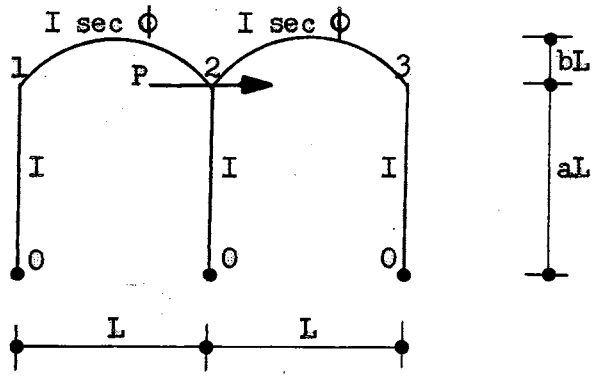
TABLE 3-2-1 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.07333	-.05870	-.04848	-.04111	-.03559
0.4	-.16226	-.14545	-.13086	-.11837	-.10769
0.5	-.20507	-.18880	-.17389	-.16048	-.14851
0.6	-.24678	-.23134	-.21667	-.20299	-.19038
0.8	-.32742	-.31384	-.30035	-.28718	-.27449
1.0	-.40524	-.39336	-.38126	-.36915	-.35714

Q23					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.06167	+.06464	+.06371	+.06168	+.05947
0.4	+.11417	+.12828	+.13582	+.13918	+.13999
0.5	+.13782	+.15626	+.16826	+.17556	+.17952
0.6	+.16035	+.18234	+.19833	+.20952	+.21698
0.8	+.20304	+.23039	+.25282	+.27086	+.28510
1.0	+.24357	+.27469	+.30185	+.32526	+.34524

Q30 = -Q32					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03833	-.02751	-.02050	-.01589	-.01276
0.4	-.09626	-.08283	-.07088	-.06082	-.05252
0.5	-.12643	-.11322	-.10062	-.08923	-.07925
0.6	-.15704	-.14433	-.13167	-.11968	-.10866
0.8	-.21921	-.20771	-.19570	-.18368	-.17197
1.0	-.28229	-.27191	-.26089	-.24954	-.23810

TWO SPAN FRAME — PARABOLIC GIRDER — TABLE 3-2-2



$$M_{ij} = PQ_{ij}L$$

$Q10 = -Q12 = Q30 = -Q32$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.04000	-.03152	-.02576	-.02167	-.01864
0.4	-.09623	-.08485	-.07531	-.06735	-.06068
0.5	-.12584	-.11407	-.10367	-.09457	-.08663
0.6	-.15605	-.14419	-.13333	-.12351	-.11466
0.8	-.21772	-.20615	-.19509	-.18462	-.17478
1.0	-.28055	-.26952	-.25871	-.24822	-.23810

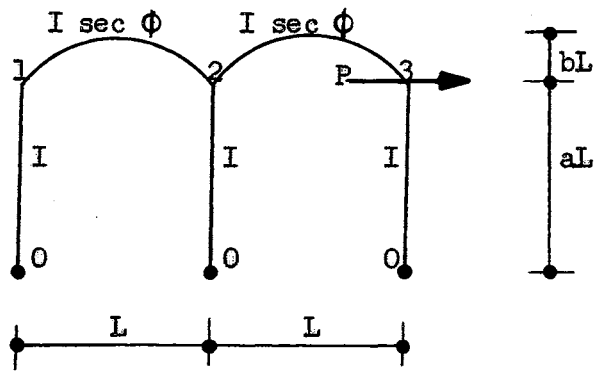
$Q21 = Q23$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.06000	+.06848	+.07424	+.07833	+.08136
0.4	+.10377	+.11515	+.12469	+.13265	+.13932
0.5	+.12416	+.13593	+.14633	+.15543	+.16337
0.6	+.14395	+.15581	+.16667	+.17649	+.18534
0.8	+.18228	+.19385	+.20491	+.21538	+.22522
1.0	+.21945	+.23048	+.24129	+.25178	+.26190

TABLE 3-2-2 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.12000	-.13696	-.14848	-.15667	-.16271
0.4	-.20755	-.23030	-.24938	-.26531	-.27863
0.5	-.24832	-.27187	-.29267	-.31087	-.32673
0.6	-.28791	-.31162	-.33333	-.35299	-.37067
0.8	-.36455	-.38769	-.40982	-.43077	-.45044
1.0	-.43890	-.46096	-.48257	-.50356	-.52381

TWO SPAN FRAME PARABOLIC GIRDER TABLE 3-2-3



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03833	-.02751	-.02050	-.01589	-.01276
0.4	-.09626	-.08283	-.07088	-.06082	-.05252
0.5	-.12643	-.11322	-.10062	-.08923	-.07925
0.6	-.15704	-.14433	-.13167	-.11968	-.10866
0.8	-.21921	-.20771	-.19570	-.18368	-.17197
1.0	-.28229	-.27191	-.26089	-.24954	-.23810

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.06167	+.06464	+.06371	+.06168	+.05947
0.4	+.11417	+.12828	+.13582	+.13918	+.13999
0.5	+.13782	+.15626	+.16826	+.17556	+.17952
0.6	+.16035	+.18234	+.19833	+.20952	+.21698
0.8	+.20304	+.23093	+.25282	+.27086	+.28510
1.0	+.24357	+.27469	+.30185	+.32526	+.34524

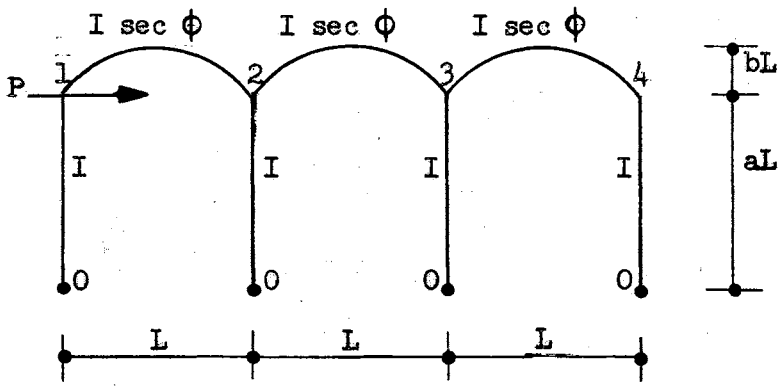
TABLE 3-2-3 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.07333	-.05870	-.04848	-.04111	-.03559
0.4	-.16226	-.14545	-.13086	-.11837	-.10769
0.5	-.20507	-.18880	-.17389	-.16048	-.14851
0.6	-.24678	-.23134	-.21667	-.20299	-.19038
0.8	-.32742	-.31384	-.30035	-.28718	-.27449
1.0	-.40524	-.39336	-.38126	-.36915	-.35714

Q23					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.01167	-.00594	-.01523	-.02056	-.02387
0.4	+.04809	+.01718	-.00496	-.02082	-.03229
0.5	+.06726	+.03254	+.00562	-.01508	-.03100
0.6	+.08643	+.04900	+.01833	-.00654	-.02660
0.8	+.12438	+.08345	+.04754	+.01632	-.01062
1.0	+.16167	+.11867	+.07941	+.04388	+.01190

Q30 = -Q32					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.08833	-.11379	-.13102	-.14299	-.15164
0.4	-.14148	-.17171	-.19826	-.22082	-.23978
0.5	-.16849	-.19798	-.22550	-.25029	-.27223
0.6	-.19618	-.22433	-.25167	-.27734	-.30096
0.8	-.25335	-.27845	-.30394	-.32914	-.35355
1.0	-.31247	-.33473	-.35785	-.38132	-.40476

THREE SPAN FRAME | PARABOLIC GIRDER | TABLE 3-3-1



$$M_{ij} = PQ_{ij}L$$

$Q10 = -Q12$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.07809	-.10796	-.12754	-.14076	-.15012
0.4	-.11456	-.15017	-.18151	-.20787	-.22972
0.5	-.13346	-.16796	-.20047	-.22970	-.25536
0.6	-.15312	-.18588	-.21811	-.24846	-.27630
0.8	-.19445	-.22344	-.25332	-.28308	-.31196
1.0	-.23774	-.26339	-.29039	-.31804	-.34577

$Q21$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.00460	-.01952	-.02599	-.02921	-.03101
0.4	+.01788	-.01487	-.03604	-.04968	-.05857
0.5	+.03211	-.00630	-.03407	-.05384	-.06785
0.6	+.04718	+.00467	-.02849	-.05390	-.07318
0.8	+.07863	+.03089	-.00993	-.04431	-.07293
1.0	+.11097	+.06022	+.01462	-.02593	-.06162

TABLE 3-3-1 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.05164	-.04492	-.03939	-.03474	-.03089
0.4	-.10989	-.10087	-.09423	-.08843	-.08313
0.5	-.13787	-.12882	-.12164	-.11549	-.10992
0.6	-.16607	-.15673	-.14910	-.14256	-.13672
0.8	-.22152	-.21219	-.20408	-.19692	-.19051
1.0	-.27593	-.26699	-.25885	-.25143	-.24463

Q23					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05623	+.06445	+.06537	+.06395	+.06190
0.4	+.09147	+.11574	+.13027	+.13812	+.14170
0.5	+.10577	+.13513	+.15571	+.16933	+.17777
0.6	+.11889	+.15206	+.17759	+.19647	+.20989
0.8	+.14288	+.18130	+.21400	+.24123	+.26344
1.0	+.16496	+.20677	+.24423	+.27735	+.30625

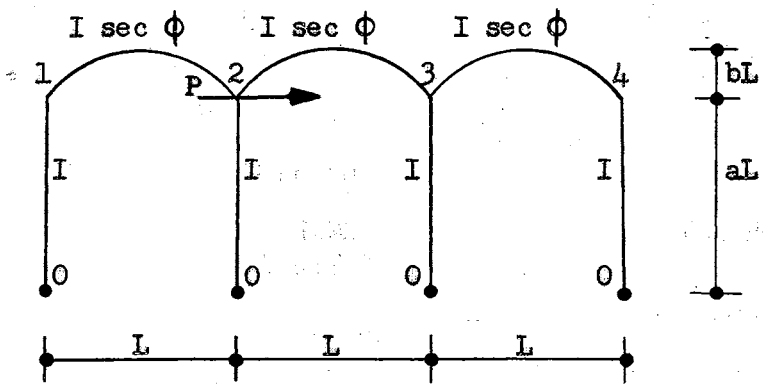
Q32					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00759	-.00239	-.00633	-.00789	-.00850
0.4	+.03097	+.01256	-.00001	-.00818	-.01334
0.5	+.04267	+.02259	+.00715	-.00413	-.01213
0.6	+.05399	+.03300	+.01563	+.00187	-.00871
0.8	+.07556	+.05378	+.03424	+.01723	+.00276
1.0	+.09600	+.07397	+.05341	+.03461	+.01769

Q30					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.04624	-.03232	-.02347	-.01786	-.01413
0.4	-.10935	-.09535	-.08149	-.06953	-.05961
0.5	-.14017	-.12726	-.11374	-.10095	-.08946
0.6	-.16953	-.15831	-.14582	-.13322	-.12118
0.8	-.22628	-.21787	-.20786	-.19692	-.18563
1.0	-.28121	-.27483	-.26697	-.25801	-.24833

TABLE 3-3-1 (Continued)

Q34						Q40 = -Q43					
a \ b	0.1	0.2	0.3	0.4	0.5	a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03864	+.03470	+.02979	+.02575	+.02263	0.2	-.02403	-.01480	-.00960	-.00664	-.00486
0.4	+.07892	+.08279	+.08152	+.07770	+.07295	0.4	-.06620	-.05361	-.04277	-.03417	-.02754
0.5	+.09749	+.10468	+.10659	+.10508	+.10159	0.5	-.08852	-.07596	-.06415	-.05386	-.04526
0.6	+.11554	+.12531	+.13019	+.13134	+.12990	0.6	-.11128	-.09908	-.08697	-.07576	-.06580
0.8	+.15071	+.16409	+.17361	+.17969	+.18287	0.8	-.15775	-.14650	-.13474	-.12308	-.11190
1.0	+.18521	+.20086	+.21356	+.22341	+.23064	1.0	-.20510	-.19479	-.18379	-.17252	-.16127

THREE SPAN FRAME PARABOLIC GIRDER TABLE 3-3-2



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02856	-.02418	-.02088	-.01825	-.01612
0.4	-.07001	-.06038	-.05502	-.05068	-.04696
0.5	-.08896	-.08072	-.07436	-.06918	-.06476
0.6	-.11108	-.10218	-.09497	-.08899	-.08387
0.8	-.15682	-.14731	-.13904	-.13182	-.12547
1.0	-.20384	-.19430	-.18560	-.17771	-.17052

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04183	+.05137	+.05919	+.06513	+.06964
0.4	+.07166	+.07879	+.08706	+.09549	+.10351
0.5	+.08720	+.09285	+.09987	+.10765	+.11564
0.6	+.10301	+.10744	+.11318	+.11991	+.12723
0.8	+.13512	+.13782	+.14150	+.14610	+.15151
1.0	+.16764	+.16922	+.17148	+.17444	+.17812

TABLE 3-3-2 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.09576	-.11960	-.13575	-.14693	-.15500
0.4	-.15121	-.17962	-.20496	-.22673	-.24512
0.5	-.17748	-.20516	-.23136	-.25528	-.27664
0.6	-.20342	-.22987	-.25585	-.28053	-.30350
0.8	-.25481	-.27855	-.30275	-.32684	-.35039
1.0	-.30580	-.32708	-.34912	-.37155	-.39405

Q23					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05393	+.06823	+.07656	+.08180	+.08535
0.4	+.07955	+.10083	+.11790	+.13123	+.14161
0.5	+.09028	+.11231	+.13149	+.14763	+.16101
0.6	+.10042	+.12244	+.14267	+.16062	+.17627
0.8	+.11968	+.14072	+.16124	+.18073	+.19887
1.0	+.13816	+.15786	+.17765	+.19710	+.21593

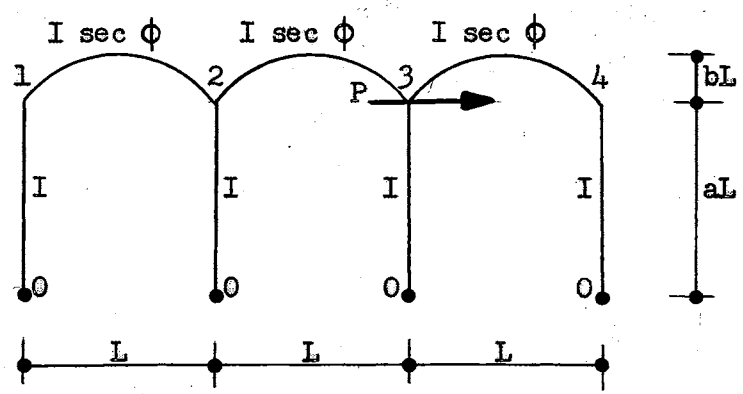
Q32					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00565	-.00617	-.01180	-.01468	-.01625
0.4	+.02657	+.00483	-.01078	-.02167	-.02923
0.5	+.03728	+.01293	-.00629	-.02103	-.03217
0.6	+.04776	+.02164	-.00029	-.01824	-.03269
0.8	+.06806	+.03968	+.01416	-.00843	-.02813
1.0	+.08758	+.05786	+.03013	+.00458	-.01875

Q30					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.04892	-.03764	-.02997	-.02469	-.02092
0.4	-.11119	-.09962	-.08868	-.07901	-.07072
0.5	-.14096	-.13056	-.11998	-.10990	-.10066
0.6	-.16994	-.16073	-.15093	-.14111	-.13166
0.8	-.22617	-.21895	-.21091	-.20240	-.19367
1.0	-.28082	-.27502	-.26848	-.26135	-.25383

TABLE 3-3-2 (Continued)

Q34						Q40 = -Q43					
a \ b	0.1	0.2	0.3	0.4	0.5	a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04327	+.04381	+.04177	+.03937	+.03716	0.2	-.02627	-.01858	-.01340	-.01013	-.00796
0.4	+.08460	+.09479	+.09946	+.10069	+.09995	0.4	-.06759	-.06038	-.05134	-.04358	-.03720
0.5	+.10368	+.11763	+.12627	+.13093	+.13282	0.5	-.09262	-.08356	-.07430	-.06564	-.05794
0.6	+.12217	+.13910	+.15122	+.15935	+.16435	0.6	-.11556	-.10722	-.09825	-.08937	-.08097
0.8	+.15812	+.17926	+.19676	+.21082	+.22179	0.8	-.16218	-.15519	-.14730	-.13894	-.13047
1.0	+.19324	+.21716	+.23834	+.25678	+.27258	1.0	-.20952	-.20360	-.19680	-.18939	-.18160

THREE SPAN FRAME | PARABOLIC GIRDER | TABLE 3-3-3



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02676	-.01858	-.01340	-.01013	-.00796
0.4	-.06759	-.06038	-.05134	-.04358	-.03720
0.5	-.09262	-.08356	-.07430	-.06564	-.05794
0.6	-.11556	-.10722	-.09825	-.08937	-.08097
0.8	-.16218	-.15519	-.14730	-.13894	-.13047
1.0	-.20952	-.20360	-.19680	-.18939	-.18160

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04327	+.04381	+.04177	+.03937	+.03716
0.4	+.08460	+.09479	+.09946	+.10069	+.09995
0.5	+.10368	+.11763	+.12627	+.13093	+.13282
0.6	+.12217	+.13910	+.15122	+.15935	+.16435
0.8	+.15812	+.17926	+.19676	+.21082	+.22179
1.0	+.19324	+.21716	+.23834	+.25678	+.27258

TABLE 3-3-3 (Continued)

Q20

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.04892	-.03764	-.02997	-.02469	-.02092
0.4	-.11119	-.09962	-.08868	-.07901	-.07072
0.5	-.14096	-.13056	-.11998	-.10990	-.10066
0.6	-.16994	-.16073	-.15093	-.14111	-.13166
0.8	-.22617	-.21895	-.21091	-.20240	-.19367
1.0	-.28082	-.27502	-.26848	-.26135	-.25383

Q23

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00565	-.00617	-.01180	-.01468	-.01625
0.4	+.02657	+.00483	-.01078	-.02167	-.02923
0.5	+.03728	+.01293	-.00629	-.02103	-.03217
0.6	+.04776	+.02164	-.00029	-.01824	-.03269
0.8	+.06806	+.03968	+.01416	-.00843	-.02813
1.0	+.08758	+.05786	+.03013	+.00458	-.01875

Q32

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05393	+.06823	+.07656	+.08180	+.08535
0.4	+.07955	+.10083	+.11790	+.13123	+.14161
0.5	+.09028	+.11231	+.13149	+.14763	+.16101
0.6	+.10042	+.12244	+.14267	+.16062	+.17627
0.8	+.11968	+.14072	+.16124	+.18073	+.19887
1.0	+.13816	+.15786	+.17765	+.19710	+.21593

Q30

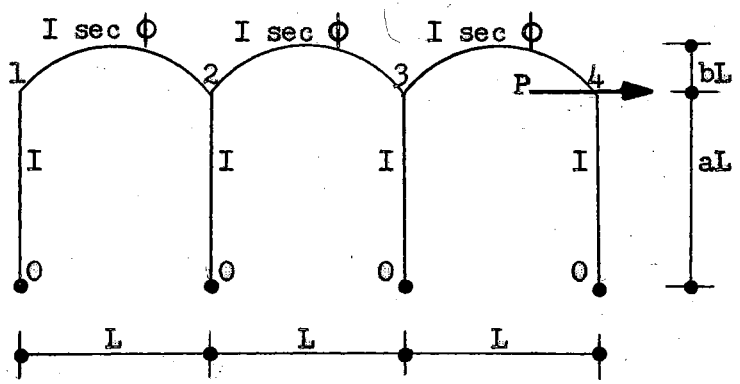
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.09576	-.11960	-.13575	-.14693	-.15500
0.4	-.15121	-.17962	-.20496	-.22673	-.24512
0.5	-.17748	-.20516	-.23136	-.25528	-.27664
0.6	-.20342	-.22987	-.25585	-.28053	-.30350
0.8	-.25481	-.27855	-.30275	-.32684	-.35039
1.0	-.30580	-.32708	-.34912	-.37155	-.39405

TABLE 3-3-3 (Continued)

Q34					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04183	+.05137	+.05919	+.06513	+.06964
0.4	+.07166	+.07879	+.08706	+.09549	+.10351
0.5	+.08720	+.09285	+.09987	+.10765	+.11564
0.6	+.10301	+.10744	+.11318	+.11991	+.12723
0.8	+.13512	+.13782	+.14150	+.14610	+.15151
1.0	+.16764	+.16922	+.17148	+.17444	+.17812

Q40 = -Q43					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02856	-.02418	-.02088	-.01825	-.01612
0.4	-.07001	-.06038	-.05502	-.05068	-.04696
0.5	-.08896	-.08072	-.07436	-.06918	-.06476
0.6	-.11108	-.10218	-.09497	-.08899	-.08387
0.8	-.15682	-.14731	-.13904	-.13182	-.12547
1.0	-.20384	-.19430	-.18560	-.17771	-.17052

THREE SPAN FRAME PARABOLIC GIRDER TABLE 3-3-4



$$M_{ij} = PQ_{ij}L$$

$Q10 = -Q12$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02403	-.01480	-.00960	-.00664	-.00486
0.4	-.06620	-.05361	-.04277	-.03417	-.02754
0.5	-.08852	-.07596	-.06415	-.05386	-.04526
0.6	-.11128	-.09908	-.08697	-.07576	-.06580
0.8	-.15775	-.14650	-.13474	-.12308	-.11190
1.0	-.20510	-.19479	-.18379	-.17252	-.16127

$Q21$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03864	+.03470	+.02979	+.02575	+.02263
0.4	+.07892	+.08279	+.08152	+.07770	+.07295
0.5	+.09749	+.10468	+.10659	+.10508	+.10159
0.6	+.11554	+.12531	+.13015	+.13134	+.12990
0.8	+.15071	+.16409	+.17361	+.17969	+.18287
1.0	+.18521	+.20086	+.21356	+.22341	+.23064

TABLE 3-3-4 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.04624	-.03232	-.02347	-.01786	-.01413
0.4	-.10935	-.09535	-.08149	-.06953	-.05961
0.5	-.14017	-.12726	-.11374	-.10095	-.08946
0.6	-.16953	-.15831	-.14582	-.13322	-.12118
0.8	-.22628	-.21787	-.20786	-.19692	-.18563
1.0	-.28121	-.27483	-.26697	-.25801	-.24833

Q23					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00759	-.00239	-.00633	-.00789	-.00850
0.4	+.03097	+.01256	-.00001	-.00818	-.01334
0.5	+.04267	+.02259	+.00715	-.00413	-.01213
0.6	+.05399	+.03300	+.01563	+.00187	-.00871
0.8	+.07556	+.05378	+.03424	+.01723	+.00276
1.0	+.09600	+.07397	+.05341	+.03461	+.01769

Q32					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05623	+.06445	+.06537	+.06395	+.06190
0.4	+.09147	+.11574	+.13027	+.13812	+.14170
0.5	+.10577	+.13513	+.15571	+.16933	+.17777
0.6	+.11889	+.15206	+.17759	+.19647	+.20989
0.8	+.14288	+.18130	+.21400	+.24123	+.26344
1.0	+.16496	+.20677	+.24423	+.27735	+.30625

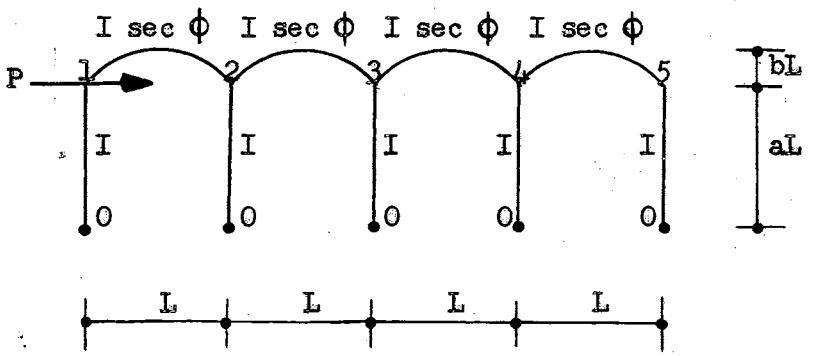
Q30					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.05164	-.04492	-.03939	-.03474	-.03089
0.4	-.10989	-.10087	-.09423	-.08843	-.08313
0.5	-.13787	-.12882	-.12164	-.11549	-.10992
0.6	-.16607	-.15673	-.14910	-.14256	-.13672
0.8	-.22152	-.21219	-.20408	-.19692	-.19051
1.0	-.27593	-.26699	-.25885	-.25143	-.24463

TABLE 3-3-4 (Continued)

Q34					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.00460	-.01952	-.02599	-.02921	-.03101
0.4	+.01788	-.01487	-.03604	-.04968	-.05857
0.5	+.03211	-.00630	-.03407	-.05384	-.06785
0.6	+.04718	+.00467	-.02849	-.05390	-.07318
0.8	+.07863	+.03089	-.00993	-.04431	-.07293
1.0	+.11097	+.06022	+.01462	-.02593	-.06162

Q40 = -Q43					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.07809	-.10796	-.12754	-.14076	-.15012
0.4	-.11456	-.15017	-.18151	-.20787	-.22972
0.5	-.11346	-.16796	-.20047	-.22970	-.25536
0.6	-.15312	-.18588	-.21811	-.24846	-.27630
0.8	-.19445	-.22344	-.25332	-.28308	-.31196
1.0	-.23774	-.26339	-.29039	-.31804	-.34577

FOUR SPAN FRAME — PARABOLIC GIRDER — TABLE 3-4-1



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.07369	-.10618	-.12675	-.14036	-.14988
0.4	-.10022	-.14008	-.17479	-.20343	-.22674
0.5	-.11400	-.15266	-.18905	-.22136	-.24929
0.6	-.12857	-.16523	-.20147	-.23538	-.26614
0.8	-.15970	-.19200	-.22565	-.25921	-.29168
1.0	-.19284	-.22128	-.25161	-.28284	-.31419

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.01169	-.02372	-.02846	-.03077	-.03207
0.4	+.00065	-.03052	-.04890	-.06052	-.06647
0.5	+.01039	-.02759	-.05320	-.07021	-.08153
0.6	+.02117	-.02187	-.05373	-.07684	-.09341
0.8	+.04433	-.00532	-.04646	-.07990	-.10671
1.0	+.06851	+.01500	-.03214	-.07304	-.10816

TABLE 3-4-1 (Continued)

Q20

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03325	-.04105	-.03745	-.03366	-.03023
0.4	-.08619	-.08340	-.08173	-.07960	-.07682
0.5	-.10809	-.10404	-.10204	-.10034	-.09829
0.6	-.13009	-.12501	-.12226	-.12042	-.18870
0.8	-.17395	-.16764	-.16345	-.16060	-.15850
1.0	-.21741	-.21066	-.20555	-.20169	-.19875

Q23

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05494	+.06477	+.06591	+.06443	+.06230
0.4	+.08553	+.11392	+.13063	+.13941	+.14339
0.5	+.09770	+.13163	+.15524	+.17055	+.17983
0.6	+.10892	+.14687	+.17600	+.19726	+.21212
0.8	+.12963	+.17297	+.20991	+.24050	+.26522
1.0	+.14890	+.19567	+.23768	+.27473	+.30691

Q32

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.00243	-.01003	-.01167	-.01173	-.01137
0.4	+.01331	-.00748	-.01920	-.02519	-.02797
0.5	+.02307	-.00140	-.01789	-.02819	-.03424
0.6	+.03315	+.00635	-.01386	-.02818	-.03785
0.8	+.05365	+.02440	-.00054	-.02098	-.03716
1.0	+.07420	+.04393	+.01649	-.00771	-.02855

Q30

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03552	-.02620	-.01982	-.01552	-.01253
0.4	-.08179	-.07257	-.06362	-.05563	-.04877
0.5	-.10396	-.09574	-.08715	-.07887	-.07124
0.6	-.12563	-.11840	-.11049	-.10245	-.09466
0.8	-.16798	-.16237	-.15591	-.14896	-.14178
1.0	-.20952	-.20504	-.19980	-.19400	-.18779

TABLE 3-4-1 (Continued)

Q34					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03795	+.03623	+.03149	+.02725	+.02391
0.4	+.06849	+.08004	+.08282	+.08083	+.07673
0.5	+.08089	+.09714	+.10505	+.10707	+.10548
0.6	+.09247	+.11205	+.12434	+.13064	+.13251
0.8	+.11433	+.13796	+.15646	+.16994	+.17894
1.0	+.13532	+.16111	+.18331	+.20171	+.21635

Q43					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00510	-.00139	-.00309	-.00339	-.00330
0.4	+.02319	+.00850	-.00035	-.00517	-.00769
0.5	+.03262	+.01605	+.00434	-.00327	-.00791
0.6	+.04188	+.02417	+.01036	+.00032	-.00662
0.8	+.05983	+.04091	+.02451	+.01092	+.00006
1.0	+.07714	+.05761	+.03978	+.02401	+.01039

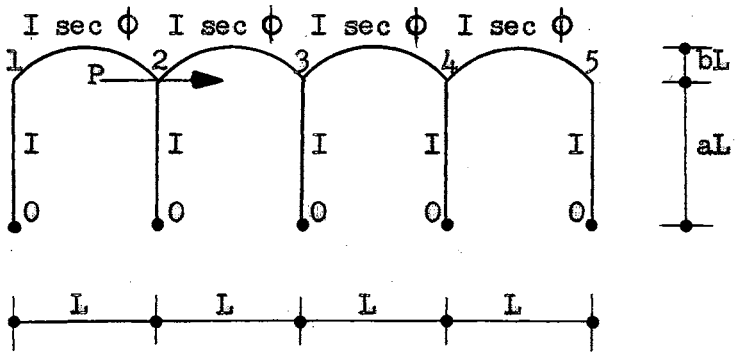
Q40					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02127	-.01821	-.01133	-.00762	-.00547
0.4	-.08225	-.06646	-.05227	-.04104	-.03254
0.5	-.10667	-.09228	-.07768	-.06466	-.05375
0.6	-.13037	-.11759	-.10362	-.09008	-.07782
0.8	-.17615	-.16618	-.15441	-.14186	-.12930
1.0	-.22049	-.21256	-.20293	-.19215	-.18075

Q45					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.02617	+.01960	+.01442	+.01101	+.00877
0.4	+.05905	+.05796	+.05262	+.04690	+.04013
0.5	+.07405	+.07623	+.07334	+.06793	+.06167
0.6	+.08849	+.09341	+.09325	+.08976	+.08445
0.8	+.11633	+.12528	+.12990	+.13094	+.12923
1.0	+.14335	+.15496	+.16314	+.16814	+.17036

TABLE 3-4-1 (Continued)

Q50 = -Q54					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.01627	-.00836	-.00465	-.00284	-.00188
0.4	-.04954	-.03750	-.02759	-.02029	-.01514
0.5	-.06728	-.05526	-.04407	-.03476	-.02743
0.6	-.08535	-.07377	-.06217	-.05166	-.04268
0.8	-.12220	-.11180	-.10057	-.08937	-.07876
1.0	-.15974	-.15044	-.14011	-.12932	-.11851

FOUR SPAN FRAME — PARABOLIC GIRDER — TABLE 3-4-2



$$M_{ij} = PQ_{ij}L$$

$Q10 = -Q12$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02369	-.02196	-.01978	-.01764	-.01575
0.4	-.05259	-.04919	-.04709	-.04512	-.04301
0.5	-.06881	-.06418	-.06139	-.05924	-.05718
0.6	-.08584	-.08019	-.07657	-.07393	-.07169
0.8	-.12144	-.11456	-.10946	-.10559	-.10251
1.0	-.15835	-.15095	-.14494	-.14005	-.13602

$Q21$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03399	+.04615	+.05576	+.06277	+.06792
0.4	+.05362	+.06142	+.07189	+.08279	+.09302
0.5	+.06471	+.06981	+.07816	+.08812	+.09853
0.6	+.07627	+.07921	+.08528	+.09353	+.10298
0.8	+.10018	+.10010	+.10244	+.10698	+.11329
1.0	+.12464	+.12286	+.12247	+.12406	+.12729

TABLE 3-4-2 (Continued)

Q20

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.08649	-.11477	-.13307	-.14530	-.15392
0.4	-.12695	-.16022	-.19022	-.21565	-.23676
0.5	-.14663	-.17834	-.20911	-.23722	-.26210
0.6	-.16644	-.19611	-.22617	-.25506	-.28191
0.8	-.20635	-.23212	-.25930	-.28690	-.31416
1.0	-.24650	-.26907	-.29322	-.31833	-.34391

Q23

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05249	+.06863	+.07731	+.08253	+.08601
0.4	+.07333	+.09881	+.11833	+.13286	+.14374
0.5	+.08193	+.10853	+.13095	+.14909	+.16357
0.6	+.09017	+.11691	+.14089	+.16153	+.17892
0.8	+.10617	+.13203	+.15684	+.17991	+.20087
1.0	+.12186	+.14641	+.17074	+.19427	+.21662

Q32

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.00544	-.01569	-.01920	-.02048	-.02092
0.4	+.00807	-.01742	-.03340	-.04300	-.04863
0.5	+.01696	-.01303	-.03473	-.04973	-.05982
0.6	+.02633	-.00672	-.03291	-.05283	-.06760
0.8	+.04571	+.00904	-.02308	-.05047	-.07333
1.0	+.06543	+.02684	-.00867	-.04078	-.06937

Q30

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03707	-.03002	-.02491	-.02115	-.01831
0.4	-.08176	-.07434	-.06760	-.06161	-.05634
0.5	-.10347	-.09646	-.08980	-.08359	-.07787
0.6	-.12483	-.11829	-.11189	-.10573	-.09990
0.8	-.16684	-.16116	-.15542	-.14972	-.14410
1.0	-.20823	-.20324	-.19814	-.19298	-.18779

TABLE 3-4-2 (Continued)

Q34

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04250	+.04571	+.04412	+.04164	+.03924
0.4	+.07369	+.09176	+.10100	+.10460	+.10497
0.5	+.08650	+.10949	+.12453	+.13331	+.13770
0.6	+.09849	+.12500	+.14479	+.15857	+.16750
0.8	+.12113	+.15212	+.17850	+.20019	+.21743
1.0	+.14279	+.17640	+.20681	+.23376	+.25717

Q43

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00581	-.00159	-.00419	-.00507	-.00533
0.4	+.02499	+.01033	-.00047	-.00568	-.00936
0.5	+.03477	+.01877	+.00643	-.00243	-.00853
0.6	+.04429	+.02763	+.01365	+.00263	-.00572
0.8	+.06261	+.04543	+.02972	+.01593	+.00419
1.0	+.08016	+.06283	+.04636	+.03113	+.01734

Q40

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03475	-.02283	-.01579	-.01160	-.00894
0.4	-.08677	-.07458	-.06250	-.05217	-.04380
0.5	-.11133	-.10112	-.08957	-.07844	-.06848
0.6	-.13506	-.12675	-.11649	-.10568	-.09525
0.8	-.18073	-.17540	-.16798	-.15922	-.14980
1.0	-.22486	-.22149	-.21634	-.20979	-.20225

Q45

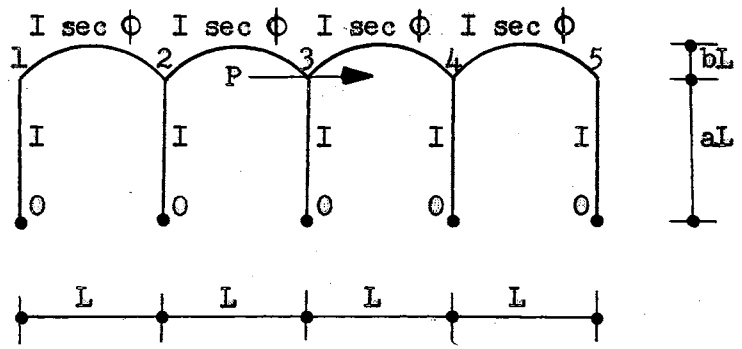
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.02895	+.02443	+.01998	+.01667	+.01428
0.4	+.06178	+.06424	+.06203	+.05785	+.05316
0.5	+.07655	+.08280	+.08314	+.08086	+.07701
0.6	+.09075	+.09913	+.10284	+.10305	+.10096
0.8	+.19902	+.12998	+.13826	+.14330	+.14561
1.0	+.14470	+.15846	+.16997	+.17866	+.18491

TABLE 3-4-2 (Continued)

$Q_{50} = -Q_{54}$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.01801	-.01042	-.00644	-.00430	-.00307
0.4	-.05193	-.04167	-.03259	-.02546	-.02009
0.5	-.06975	-.05990	-.05013	-.04152	-.03436
0.6	-.08784	-.07865	-.06889	-.05959	-.05125
0.8	-.12464	-.11676	-.10784	-.09857	-.08943
1.0	-.16207	-.15525	-.14738	-.13885	-.13002

FOUR SPAN FRAME — PARABOLIC GIRDER — TABLE 3-4-3



$$M_{ij} = PQ_{ij}L$$

$Q10 = -Q12 = Q50 = -Q54$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02104	-.01529	-.01143	-.00886	-.00710
0.4	-.05422	-.04749	-.04120	-.03569	-.03103
0.5	-.07174	-.06530	-.05886	-.05281	-.04733
0.6	-.08960	-.08357	-.07727	-.07108	-.06520
0.8	-.12611	-.12085	-.11516	-.10926	-.10334
1.0	-.16335	-.15873	-.15368	-.14833	-.14280

$Q21 = Q45$

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03407	+.03610	+.03567	+.03449	+.03315
0.4	+.06563	+.07478	+.08005	+.08267	+.08357
0.5	+.08036	+.09221	+.10040	+.10572	+.10889
0.6	+.09468	+.10869	+.11941	+.12731	+.13292
0.8	+.12248	+.13966	+.15430	+.16654	+.17661
1.0	+.14956	+.16892	+.18631	+.20177	+.21538

TABLE 3-4-3 (Continued)

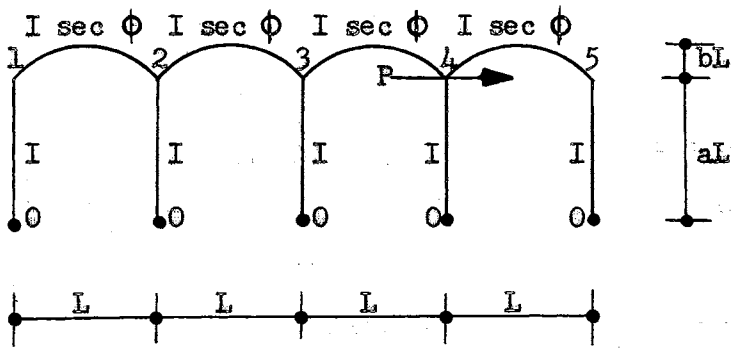
Q20 = Q40					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03803	-.03053	-.02520	-.02133	-.01843
0.4	-.08567	-.07729	-.06983	-.06331	-.05765
0.5	-.10899	-.10098	-.09348	-.08658	-.08031
0.6	-.13193	-.12442	-.11713	-.11020	-.10370
0.8	-.17682	-.17032	-.16377	-.15728	-.15092
1.0	-.22069	-.21506	-.20929	-.20344	-.19757

Q23 = Q43					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00396	-.00557	-.01047	-.01316	-.01472
0.4	+.02004	+.00251	-.01022	-.01937	-.02592
0.5	+.02863	+.00878	-.00692	-.01914	-.02857
0.6	+.03725	+.01573	-.00227	-.01711	-.02922
0.8	+.05434	+.03066	+.00947	-.00926	-.02569
1.0	+.07113	+.04615	+.02299	+.00167	-.01780

Q32 = Q34					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04093	+.05418	+.06336	+.06981	+.07447
0.4	+.06011	+.07522	+.08898	+.10100	+.11132
0.5	+.06927	+.08371	+.09766	+.11061	+.12236
0.6	+.07847	+.09202	+.10559	+.11872	+.13110
0.8	+.09707	+.10884	+.12107	+.13346	+.14574
1.0	+.11596	+.12620	+.13703	+.14823	+.15962

Q30					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.08185	-.10836	-.12673	-.13961	-.14895
0.4	-.12021	-.15044	-.17795	-.20200	-.22264
0.5	-.13854	-.16743	-.19533	-.22123	-.24471
0.6	-.15693	-.18403	-.21119	-.23743	-.26220
0.8	-.19414	-.21767	-.24215	-.26692	-.29148
1.0	-.23192	-.25241	-.27406	-.29646	-.31925

FOUR SPAN FRAME | PARABOLIC GIRDER | TABLE 3-4-4



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.01801	-.01042	-.00644	-.00430	-.00307
0.4	-.05193	-.04167	-.03259	-.02546	-.02009
0.5	-.06975	-.05990	-.05013	-.04152	-.03436
0.6	-.08784	-.07865	-.06889	-.05959	-.05125
0.8	-.12464	-.11676	-.10784	-.09857	-.08943
1.0	-.16207	-.15525	-.14738	-.13885	-.13002

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.02895	+.02443	+.01998	+.01667	+.01428
0.4	+.06178	+.06424	+.06203	+.05785	+.05316
0.5	+.07655	+.08280	+.08314	+.08086	+.07701
0.6	+.09075	+.09913	+.10284	+.10305	+.10096
0.8	+.11902	+.12998	+.13826	+.14330	+.14561
1.0	+.14470	+.15846	+.16997	+.17866	+.18491

TABLE 3-4-4 (Continued)

Q20

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03475	-.02283	-.01579	-.01160	-.00894
0.4	-.08677	-.07458	-.06250	-.05217	-.04380
0.5	-.11133	-.10112	-.08957	-.07844	-.06848
0.6	-.13506	-.12675	-.11649	-.10568	-.09525
0.8	-.18073	-.17540	-.16798	-.15922	-.14980
1.0	-.22486	-.22149	-.21634	-.20979	-.20225

Q23

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00581	-.00159	-.00419	-.00507	-.00533
0.4	+.02499	+.01033	+.00047	-.00568	-.00936
0.5	+.03477	+.01877	+.00643	-.00243	-.00853
0.6	+.04429	+.02763	+.01365	+.00263	-.00572
0.8	+.06261	+.04543	+.02972	+.01593	+.00419
1.0	+.08016	+.06283	+.04636	+.03113	+.01734

Q32

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.04250	+.04571	+.04412	+.04164	+.03924
0.4	+.07369	+.09176	+.10100	+.10460	+.10497
0.5	+.08650	+.10949	+.12453	+.13331	+.13770
0.6	+.09849	+.12500	+.14479	+.15857	+.16750
0.8	+.12113	+.15212	+.17850	+.20019	+.21743
1.0	+.14279	+.17640	+.20681	+.23376	+.25717

Q30

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03707	-.03002	-.02491	-.02115	-.01831
0.4	-.08176	-.07434	-.06760	-.06161	-.05634
0.5	-.10347	-.09646	-.08980	-.08359	-.07787
0.6	-.12483	-.11829	-.11189	-.10573	-.09990
0.8	-.16684	-.16116	-.15542	-.14972	-.14410
1.0	-.20823	-.20324	-.19814	-.19298	-.18779

TABLE 3-4-4 (Continued)

Q34

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.00544	-.01569	-.01920	-.02048	-.02092
0.4	+.00807	-.01742	-.03340	-.04300	-.04863
0.5	+.01696	-.01303	-.03473	-.04973	-.05982
0.6	+.02633	-.00672	-.03291	-.05283	-.06760
0.8	+.04571	+.00904	-.02308	-.05047	-.07333
1.0	+.06543	+.02684	-.00867	-.04078	-.06937

Q43

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05249	+.06863	+.07731	+.08253	+.08601
0.4	+.07333	+.09881	+.11833	+.13286	+.14374
0.5	+.08193	+.10853	+.13095	+.14909	+.16357
0.6	+.09017	+.11691	+.14089	+.16153	+.17892
0.8	+.10617	+.13203	+.15684	+.17991	+.20087
1.0	+.12186	+.14641	+.17074	+.19427	+.21662

Q40

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.08649	-.11477	-.13307	-.14530	-.15392
0.4	-.12695	-.16022	-.19022	-.21565	-.23676
0.5	-.14663	-.17834	-.20911	-.23722	-.26210
0.6	-.16644	-.19611	-.22617	-.25506	-.28191
0.8	-.20635	-.23212	-.25930	-.28690	-.31416
1.0	-.24650	-.29607	-.29322	-.31833	-.34391

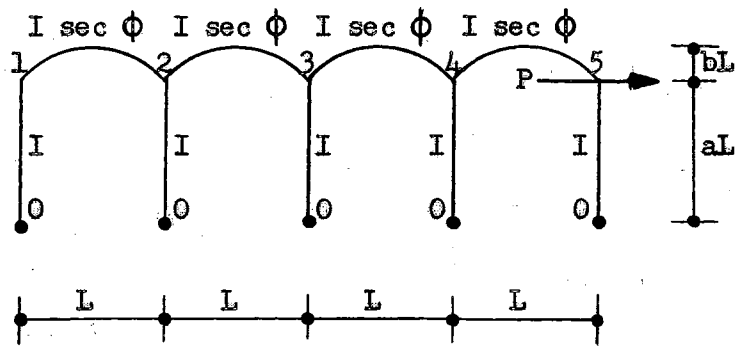
Q45

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03399	+.04615	+.05576	+.06277	+.06792
0.4	+.05362	+.06142	+.07189	+.08279	+.09302
0.5	+.06471	+.06981	+.07816	+.08812	+.09853
0.6	+.07627	+.07921	+.08528	+.09353	+.10298
0.8	+.10018	+.10010	+.10244	+.10698	+.11329
1.0	+.12464	+.12286	+.12247	+.12406	+.12729

TABLE 3-4-4 (Continued)

Q50 = -Q54					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02369	-.02196	-.01978	-.01764	-.01575
0.4	-.05259	-.04919	-.04709	-.04512	-.04301
0.5	-.06881	-.06418	-.06139	-.05924	-.05718
0.6	-.08584	-.08019	-.07657	-.07393	-.07169
0.8	-.12144	-.11456	-.10946	-.10559	-.10251
1.0	-.15835	-.15095	-.14494	-.14005	-.13602

FOUR SPAN FRAME PARABOLIC GIRDER TABLE 3-4-5



$$M_{ij} = PQ_{ij}L$$

Q10 = -Q12

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.01627	-.00836	-.00465	-.00284	-.00188
0.4	-.04954	-.03750	-.02759	-.02029	-.01514
0.5	-.06728	-.05526	-.04407	-.03476	-.02743
0.6	-.08535	-.07377	-.06217	-.05166	-.04268
0.8	-.12220	-.11180	-.10057	-.08937	-.07876
1.0	-.15974	-.15044	-.14011	-.12932	-.11851

Q21

a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.02617	+.01960	+.01442	+.01101	+.00877
0.4	+.05905	+.05796	+.05262	+.04690	+.04013
0.5	+.07405	+.07623	+.07334	+.06793	+.06167
0.6	+.08849	+.09341	+.09325	+.08976	+.08445
0.8	+.11633	+.12528	+.12990	+.13094	+.12923
1.0	+.14335	+.15496	+.16314	+.16814	+.17036

TABLE 3-4-5 (Continued)

Q20					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.02127	-.01821	-.01133	-.00762	-.00547
0.4	-.08225	-.06646	-.05227	-.04104	-.03254
0.5	-.10667	-.09228	-.07768	-.06646	-.05375
0.6	-.13037	-.11759	-.10362	-.09008	-.07782
0.8	-.17615	-.16618	-.15441	-.14186	-.12930
1.0	-.22049	-.21256	-.20293	-.19215	-.18075

Q23					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.00510	-.00139	-.00309	-.00339	-.00330
0.4	+.02319	+.00850	-.00035	-.00517	-.00769
0.5	+.03262	+.01605	+.00434	-.00327	-.00791
0.6	+.04188	+.02417	+.01036	+.00032	-.00662
0.8	+.05983	+.04091	+.02451	+.01092	+.00006
1.0	+.07714	+.05761	+.03978	+.02401	+.01039

Q32					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.03795	+.03623	+.03149	+.02725	+.02391
0.4	+.06849	+.08004	+.08282	+.08083	+.07673
0.5	+.08089	+.09714	+.10505	+.10707	+.10548
0.6	+.09247	+.11205	+.12434	+.13064	+.13251
0.8	+.11433	+.13796	+.15646	+.16994	+.17894
1.0	+.13532	+.16111	+.18331	+.20171	+.21635

Q30					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03552	-.02620	-.01982	-.01552	-.01253
0.4	-.08179	-.07257	-.06362	-.05563	-.04877
0.5	-.10396	-.09574	-.08715	-.07887	-.07124
0.6	-.12563	-.11840	-.11049	-.10245	-.09466
0.8	-.16798	-.16237	-.15591	-.14896	-.14178
1.0	-.20952	-.20504	-.19980	-.19400	-.18779

TABLE 3-4-5 (Continued)

Q34					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.00243	-.01003	-.01167	-.01173	-.01137
0.4	+.01331	-.00748	-.01920	-.02519	-.02797
0.5	+.02307	-.00140	-.01789	-.02819	-.03424
0.6	+.03315	+.00635	-.01386	-.02818	-.03785
0.8	+.05365	+.02440	-.00054	-.02098	-.03716
1.0	+.07420	+.04393	+.01649	-.00771	-.02855

Q43					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	+.05494	+.06477	+.06591	+.06443	+.06230
0.4	+.08553	+.11392	+.13063	+.13941	+.14339
0.5	+.09770	+.13163	+.15524	+.17055	+.17983
0.6	+.10892	+.14687	+.17600	+.19726	+.21212
0.8	+.19263	+.17297	+.20991	+.24050	+.26522
1.0	+.14890	+.19567	+.23768	+.27473	+.30691

Q40					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.03325	-.04105	-.03745	-.03366	-.03023
0.4	-.08619	-.08340	-.08173	-.07960	-.07682
0.5	-.10809	-.10404	-.10204	-.10034	-.09829
0.6	-.13009	-.12501	-.12226	-.12042	-.11870
0.8	-.17395	-.16764	-.16345	-.16060	-.15850
1.0	-.21741	-.21066	-.20555	-.20169	-.19875

Q45					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.01169	-.02372	-.02846	-.03077	-.03207
0.4	+.00065	-.03052	-.04890	-.06052	-.06647
0.5	+.01039	-.02759	-.05320	-.07021	-.08153
0.6	+.02117	-.02187	-.05373	-.07684	-.09341
0.8	+.04433	-.00532	-.04646	-.07990	-.10671
1.0	+.06851	+.01500	-.03214	-.07304	-.10816

TABLE 3-4-5 (Continued)

Q50 = -Q54					
a \ b	0.1	0.2	0.3	0.4	0.5
0.2	-.07369	-.10618	-.12675	-.14036	-.14988
0.4	-.10022	-.14008	-.17479	-.20343	-.22674
0.5	-.11400	-.15266	-.18905	-.22136	-.24929
0.6	-.12857	-.16523	-.20147	-.23538	-.26614
0.8	-.15970	-.19200	-.22565	-.25921	-.29168
1.0	-.19284	-.22128	-.25161	-.28284	-.31419

PART IV

ILLUSTRATIVE EXAMPLES

The application of the tables of numerical coefficients to the analysis of continuous rigid frames with curved members is shown in this part of the thesis. The results obtained are then compared to those calculated by other means.

Example 1

A three span frame loaded as shown in Figure 4-1 is analyzed. The results will be compared with those calculated in Reference 2.

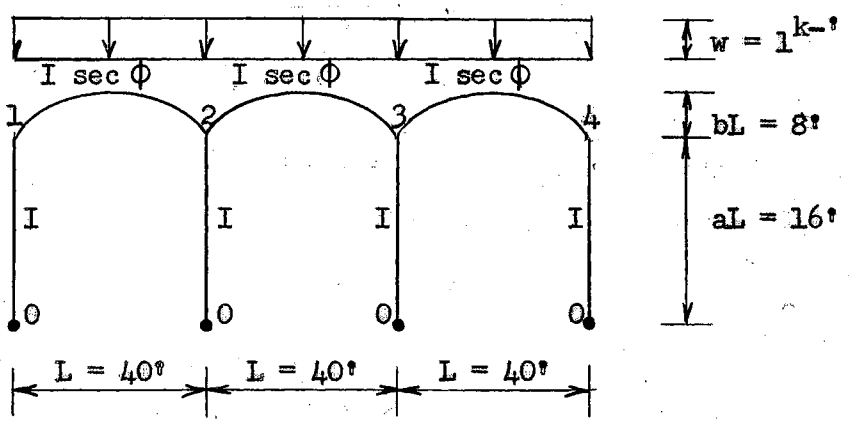


Figure 4-1

Three Span Frame

Procedure:

1. Parameters "a" and "b" :

$$a = \frac{aL}{L} = \frac{16}{40} = 0.4$$

$$b = \frac{bL}{L} = \frac{8}{40} = 0.2$$

2. Moments due to Rotation and End Moments due to Loads:

a. Moment Distribution Procedure - Joints Restrained Against Translation:

(1) Moment Distribution Constants:

$$K_{10} = K_{20} = K_{30} = K_{40} = \frac{3EI}{aL} = 7.5\,\frac{EI}{L}$$

$$K_{12} = K_{21} = K_{23} = K_{32} = K_{34} = K_{43} = \frac{9\,EI}{L}$$

$$\Sigma K_1 = \Sigma K_4 = 16.5\,\frac{EI}{L}$$

$$\Sigma K_2 = \Sigma K_3 = 25.5\,\frac{EI}{L}$$

$$D_{10} = D_{40} = \frac{K_{10}}{\Sigma K_1} = .455$$

$$D_{12} = D_{43} = \frac{K_{12}}{\Sigma K_1} = .545$$

$$D_{21} = D_{23} = D_{34} = D_{32} = \frac{K_{21}}{\Sigma K_2} = .353$$

$$D_{20} = D_{30} = \frac{K_{20}}{\Sigma K_2} = .294$$

$$C_{12} = C_{21} = C_{23} = C_{32} = C_{34} = C_{43} = \frac{\frac{-3EI}{L}}{\frac{9EI}{L}} = -.333$$

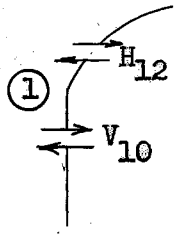
(2) Fixed End Moments:

$$FM_{12} = FM_{21} = FM_{23} = FM_{32} = FM_{34} = FM_{43} = 0$$

There are no moments due to rotation and end moments due to loads with all joints fixed against translation.

3. Unbalance of Shears (P_i) - (Equation 2-2):

a. Joint 1



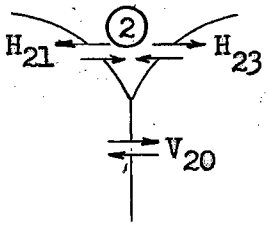
$$\vec{P_1} = V_{10} - H_{12}$$

$$V_{10} = \frac{M^L_{10}}{aL} = 0$$

$$H_{12} = FH_{12} + \frac{5}{12(2bL/3)}\left[RM_{12} - RM_{21}\right] = \frac{wL^2}{8bL} = \frac{1.0(40)^2}{8(8)} = 25^k$$

$$\vec{P_1} = -25^k$$

b. Joint 2



$$\vec{P_2} = V_{20} + H_{21} - H_{23}$$

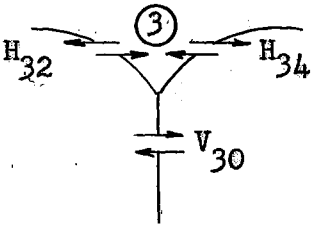
$$V_{20} = \frac{M^L_{20}}{aL} = 0$$

$$H_{21} = H_{12} = 25^k$$

$$H_{23} = FH_{23} + \frac{5}{12(2bL/3)}\left[RM_{23} - RM_{32}\right] = \frac{wL^2}{8bL} = \frac{1.0(40)^2}{8(8)} = 25^k$$

$$\vec{P_2} = 0$$

c. Joint 3



$$\vec{P_3} = V_{30} + H_{32} - H_{34}$$

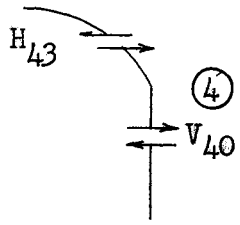
$$V_{30} = \frac{M^L_{30}}{aL} = 0$$

$$H_{32} = H_{23} = 25^k$$

$$H_{34} = FH_{34} + \frac{5}{12(2bL/3)}\left[RM_{23} - RM_{32}\right] = \frac{wL^2}{8bL} = \frac{1.0(40)^2}{8(8)} = 25^k$$

$$\vec{P_3} = 0$$

d. Joint 4



$\vec{P}_4 = V_{40} + H_{43}$

$V_{40} = 0$

$H_{43} = H_{34} = 25^k$

$\vec{P}_4 = +25^k$

4. Moment Coefficients (Q):

a. From Table 3-3-1

$Q_{10} = -0.15017$ $\quad Q_{32} = +0.01256$

$Q_{21} = -0.01487$ $\quad Q_{30} = -0.09535$

$Q_{20} = -0.10087$ $\quad Q_{34} = +0.08279$

$Q_{23} = +0.11574$ $\quad Q_{40} = -0.05361$

b. From Table 3-3-4

$Q_{10} = -0.05361$ $\quad Q_{32} = +0.11574$

$Q_{21} = +0.08279$ $\quad Q_{30} = -0.10087$

$Q_{20} = -0.09535$ $\quad Q_{34} = -0.01487$

$Q_{23} = +0.01256$ $\quad Q_{40} = -0.15017$

5. End Moments due to Unbalance of Shears ($M_{ij} = P_i Q_{ij} L$):

$M_{10} = -25 \times -0.15017 \times 40 + 25 \times 0.05361 \times 40 = +96.560^{k-'}$

$M_{21} = -25 \times -0.01487 \times 40 + 25 \times 0.08279 \times 40 = +97.660^{k-'}$

$M_{20} = -25 \times -0.10087 \times 40 + 25 \times -0.09535 \times 40 = +5.520^{k-'}$

$M_{23} = -25 \times +0.11574 \times 40 + 25 \times +0.01256 \times 40 = -103.180^{k-'}$

6. Final End Moments (Step 2 + Step 6):

The end moments of Step 2 were equal to zero; therefore, the final end moments are equal to the end moments of Step 6.

For Comparison (Units = Kip-Feet):

From tables:	From Reference 1:
$M_{10} = +96.560$	+96.544
$M_{21} = +97.660$	+97.684
$M_{20} = +5.520$	+ 5.504
$M_{23} = -103.180$	-103.152

Example 2

A two span frame loaded as shown in Figure 4-2 is analyzed. The results are compared with those obtained by moment distribution.

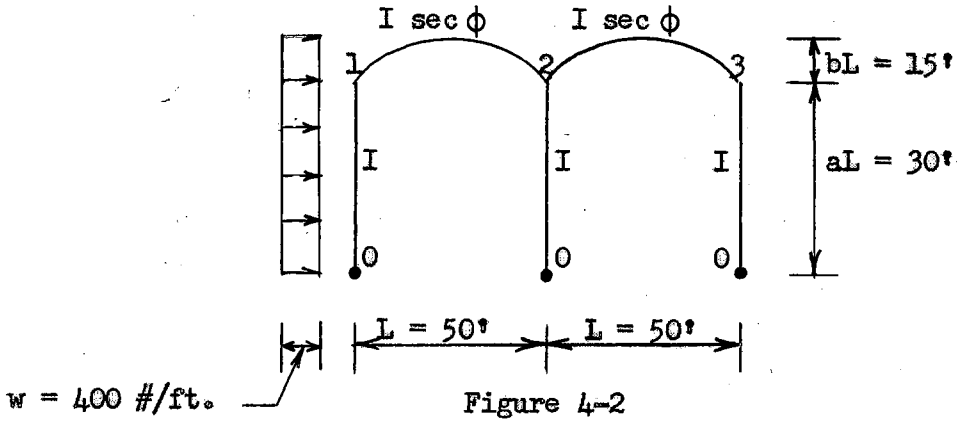


Figure 4-2

Two Span Frame

Procedure:

1. Parameters "a" and "b":

$$a = \frac{aL}{L} = \frac{30}{50} = 0.6$$

$$b = \frac{bL}{L} = \frac{15}{50} = 0.3$$

2. Moments due to Rotation and End Moments due to Loads:

a. Moment Distribution Procedure - Joints Restrained Against Translation:

(1) Moment Distribution Constants:

$$K_{10} = K_{20} = K_{30} = \frac{3EI}{aL} = 5\,\frac{EI}{L}$$

$$K_{12} = K_{21} = K_{23} = K_{32} = 9\,\frac{EI}{L}$$

$$\Sigma K_1 = \Sigma K_3 = 14\,\frac{EI}{L}$$

$$\Sigma K_2 = 23\,\frac{EI}{L}$$

$$D_{10} = D_{30} = \frac{K_{10}}{\Sigma K_1} = 0.35714$$

$$D_{12} = D_{32} = \frac{K_{12}}{\Sigma K_1} = 0.64286$$

$$D_{21} = D_{23} = \frac{K_{21}}{\Sigma K_2} = 0.39130$$

$$D_{20} = \frac{K_{20}}{\Sigma K_2} = 0.21739$$

$$C_{12} = C_{21} = C_{23} = C_{32} = \frac{\frac{-3EI}{L}}{\frac{9EI}{L}} = -0.33333$$

(2) Fixed End Moments:

$$EM_{10} = \frac{3}{2}\frac{wa^2L^2}{12} = +45^{k-'}$$

$$FM_{12} = \frac{-51wb^2L^2}{280} = -16.39286^{k-'}$$

$$FM_{21} = \frac{-19wb^2L^2}{280} = -6.10714^{k-'}$$

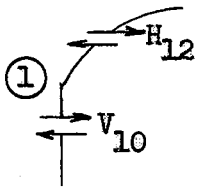
(3) Distribution of FM's due to Load:

Members	10	12	21	20	23	32	30
D's	-.35714	-.64286	-.39130	-.21739	-.39130	-.64286	-.35714
C's	0	-.33333	-.33333	0	-.33333	-.33333	0
FM's	+45.00000	-16.39286	-6.10714				
D1	-10.21675	-18.39039	+2.38972	+1.32763	+2.38972		
C1		- .79657	+6.13013			-.79657	
D2	+ .28449	+ .51208	-2.39872	-1.33263	-2.39872	+.51208	+.28449
C2		+ .79957	- .17069		- .17069	+.79957	
D3	- .28556	- .51401	+ .13358	+ .07421	+ .13358	-.51401	-.28556
C3		- .04453	+ .17133		+ .17133	-.04453	
D4	+ .01590	+ .02863	- .13408	- .07449	- .13408	+.02863	+.01590
C4		+ .04469	- .00954		- .00954	+.04469	
D5	- .01596	- .02873	+ .00747	+ .00415	+ .00747	-.02873	-.01596
C5		- .00249	+ .00958		+ .00958	-.00249	
D6	+ .00089	+ .00160	- .00750	- .00417	- .00750	+.00160	+.00089
C6		+ .00250	- .00053		- .00053	+.00250	
D7	- .00089	- .00161	+ .00041	+ .00023	+ .00041	-.00161	-.00089
C7		- .00014	+ .00054		+ .00054	-.00014	
D8	+ .00005	+ .00009	- .00042	- .00023	- .00042	+.00009	+.00005
C8		+ .00014	- .00003		- .00003	+.00014	
D9	- .00005	- .00009	+ .00002	+ .00001	+ .00002	-.00009	-.00005
C9		- .00001	+ .00003		+ .00003	-.00001	
D10		+ .00001	- .00002	- .00002	- .00002	+.00001	
RM's	-10.21788	-18.38926	+6.12128	- .00531	- .00885	+.00113	-.00113
M's	+34.78212	-34.78212	+ .01414	- .00531	- .00885	+.00113	-.00113

3. Unbalance of Shears (P_i) - (Equation 2-2):

a. Joint 1

$$\overrightarrow{P_1} = V_{10} - H_{12}$$



$$V_{10} = \frac{waL}{2} + \frac{M_{10}^{L}}{aL}$$

$$V_{10} = \frac{(.4)(30)}{2} + \frac{34.78212}{30} = 7.159^{k}$$

$$H_{12} = FH_{12} - \frac{5}{12(2bL/3)}\left[RM_{12} - RM_{21}\right]$$

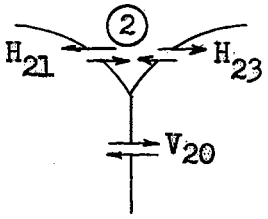
$$H_{12} = -\frac{(11)(0.4)(15)}{14} + \frac{5}{8(15)}\left[-18.38926 - 6.12128\right]$$

$$H_{12} = -4.714 - 1.021 = -5.735^{k}$$

$$\overrightarrow{P_1} = 7.159 + 5.735 = 12.894^{k}$$

b. Joint 2

$$\overrightarrow{P_2} = V_{20} + H_{21} - H_{23}$$



$$V_{20} = \frac{M_{20}^{L}}{aL} = -\frac{.00531}{30} = -.0002^{k}$$

$$H_{12} = FH_{21} + \frac{5}{12(2bL/3)}\left[RM_{12} - RM_{21}\right]$$

$$H_{12} = \frac{3wbL}{14} + \frac{5}{8bL}\left[RM_{12} - RM_{21}\right]$$

$$H_{12} = \frac{3(0.4)(15)}{14} + \frac{5}{8(15)}\left[-18.38926 - 6.12128\right]$$

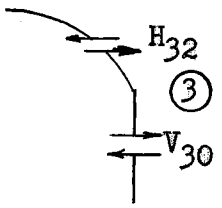
$$H_{12} = 1.286 - 1.021 = 0.265^{k}$$

$$H_{23} = \frac{5}{12(2bL/3)}\left[RM_{23} - RM_{32}\right]$$

$$H_{23} = \frac{5}{8(15)}\left[-0.00885 - 0.00113\right] = +0.0004^{k}$$

$$\overrightarrow{P_2} = -0.0002 + 0.265 - 0.0004 = +0.264^{k}$$

c. Joint 3



$\overrightarrow{P_3} = V_{30} + H_{32}$

$V_{30} = \frac{M_{30}^{L}}{aL} = \frac{-0.00113}{30} = -.00004^{k}$

$H_{32} = \frac{5}{12(2bL/3)} \left[RM_{23} - RM_{32}\right]$

$H_{32} = \frac{5}{8(15)} \left[-0.00885 - 0.00113\right] = -0.0004^{k}$

$\overrightarrow{P_3} = -0.00004 - 0.0004 = -0.00044^{k}$

4. Moment Coefficients (Q):

a. From Table 3-2-1:

$Q_{10} = -0.25167$ $\quad Q_{23} = +0.19833$

$Q_{21} = +0.01833$ $\quad Q_{30} = -0.13167$

$Q_{20} = -0.13167$

b. From Table 3-2-2:

$Q_{10} = -0.13333$ $\quad Q_{23} = +0.16667$

$Q_{21} = +0.16667$ $\quad Q_{30} = -0.13333$

$Q_{20} = -0.33333$

c. From Table 3-2-3:

$Q_{10} = -0.13167$ $\quad Q_{23} = +0.01833$

$Q_{21} = +0.19833$ $\quad Q_{30} = -0.25167$

$Q_{20} = -0.21667$

5. End Moments due to Unbalance of Shears ($M_{ij} = P_i Q_{ij} L$):

$M_{10} = 12.894 \times -0.25167 \times 50 + 0.264 \times -0.13333 \times 50 + 0.00044 \times -0.13167 \times 50$

$M_{10} = -164.012^{k-'}$

$M_{21} = 12.894 \times .01833 \times 50 + 0.264 \times 0.16667 \times 50 + 0.00044 \times 0.19833 \times 50$

$M_{21} = +14.017^{k-'}$

$M_{20} = 12.894 \times -0.21667 \times 50 + 0.264 \times -0.33333 \times 50 + 0.00044 \times -0.21667 \times 50$

$M_{20} = -144.087^{k-ft}$

$M_{23} = 12.894 \times 0.19833 \times 50 + 0.264 \times 0.16667 \times 50 + 0.00044 \times 0.01833 \times 50$

$M_{23} = +130.063^{k-ft}$

$M_{30} = 12.894 \times -0.13167 \times 50 + 0.264 \times -0.13333 \times 50 + 0.00044 \times -0.25167 \times 50$

$M_{30} = -86.648^{k-ft}$

6. Final End Moments (Step 2 + Step 6):

$M_{10} = 34.782 - 164.012 = -129.230^{k-ft}$

$M_{21} = 0.014 + 14.017 = +14.031^{k-ft}$

$M_{20} = -0.005 - 144.087 = -144.092^{k-ft}$

$M_{23} = -0.009 + 130.063 = +130.054^{k-ft}$

$M_{30} = -0.001 - 86.648 = -86.649^{k-ft}$

For Comparison (Units = Kip-Feet):

	From tables:	By Moment Distribution:
M_{10}	-129.230	-129.247
M_{21}	+14.031	+14.068
M_{20}	-144.092	-144.167
M_{23}	+130.054	+130.100
M_{30}	-86.649	-86.661

PART V

SUMMARY AND CONCLUSIONS

1. The slope deflection matrices for frames with parabolic girders and straight columns hinged at the base, loaded by unit force at joints are prepared and expressed in terms of parameters "a" and "b."

2. The evaluation of these matrices in terms of a = 0.2, 0.4, 0.5, 0.6, 0.8, 1.0 and b = 0.1, 0.2, 0.3, 0.4, 0.5 is made by use of the IBM 650 Digital Computer (machine language program).

3. The moment coefficients obtained from this machine computation are recorded in tables and rearranged for quick application.

4. The procedure of analysis in connection with these tables is described and illustrated by two examples.

5. The results obtained by the recommended procedure are compared with those from other sources.

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VITA

Robert Joe Larkin

Candidate for the Degree of

Master of Science

Title: MOMENT COEFFICIENTS FOR CONTINUOUS RIGID FRAMES WITH PARABOLIC MEMBERS

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Oklahoma City, Oklahoma, July 4, 1936, the son of W. P. and Agnes Larkin.

Education: Graduated from Choctaw High School in 1954; received Bachelor of Science degree from the Oklahoma State University, with a major in Civil Engineering, May, 1958; completed requirements for the Master of Science degree in August, 1959. Now Junior Member of A. S. C. E., member of O. S. P. E. and N. S. P. E.

Professional Experience: Civil Engineering Trainee, Tinker Air Force Base, Oklahoma City, Oklahoma; Sanitary Engineer, Public Health Service, Albuquerque, New Mexico.